Algebra Practice Problems for Precalculus and Calculus

Solve the following equations for the unknown x:

1. $5 = 7x - 16$
2. $2x - 3 = 5 - x$
3. $\frac{1}{2}(x-3) + x = 17 + 3(4-x)$
4. $\frac{5}{x} = \frac{2}{x-3}$

Multiply the indicated polynomials and simplify.

5. $(4x-1)(-3x+2)$
6. $(x-1)(x^2+x+1)$
7. $(x+1)(x^2-x+1)$
8. $(x-2)(x+2)$
9. $(x-2)(x-2)$
10. $(x^3+2x-1)(x^3-5x^2+4)$

Find the domain of each of the following functions in 11-15.

11. $f(x) = \sqrt{1+x}$
12. $f(x) = \frac{1}{1+x}$
13. $f(x) = \frac{1}{\sqrt{x}}$
14. $f(x) = \frac{1}{\sqrt{1+x}}$
15. $f(x) = \frac{1}{1+x^2}$

16. Given that $f(x) = x^2 - 3x + 4$, find and simplify $f(3)$, $f(a)$, $f(-t)$, and $f(x^2+1)$.

Factor the following quadratics

17. $x^2 - x - 20$
18. $x^2 - 10x + 21$
19. $x^2 + 10x + 16$
20. $x^2 + 8x - 105$
21. $4x^2 + 11x - 3$
22. $-2x^2 + 7x + 15$
23. $x^2 - 2$

Solve the following quadratic equations in three ways: 1) factor, 2) quadratic formula, 3) complete the square

24. $x^2 + 6x - 16 = 0$

25. $-x^2 - 3x - 2 = 0$

26. $2x^2 + 2x - 4 = 0$

Solve the following smorgasbord of equations and inequalities

27. $\sqrt{x} = \sqrt{2x - 1}$

28. $\sqrt{x^2 - 3} = \sqrt{2x}$

29. $|x - 5| = 4$

30. $2x + 4 \geq 3$

31. $-2x + 4 \geq 3$

32. $\frac{x+4}{x-3} = 2$

33. $x^2 - x - 2 > 0$

Add/Subtract the following rational expressions:

34. $\frac{x}{x+2} + \frac{3}{x-4}$

35. $\frac{x^2+1}{(x-1)(x-2)} - \frac{x^3}{x-3}$

Simplify the following rational expressions (if possible):

36. $\frac{x^2+x-2}{x^2-1}$

37. $\frac{x^2+5x+6}{x^2-3x+2}$

38.

$$\frac{\frac{x}{x+2} + 3}{\frac{x+1}{x-1}}$$

Solutions

1. Given that $5 = 7x - 16$, add 16 to both sides to get $7x = 21$. Now divide both sides by 7 to get $x = 3$. Checking, we see that $7(3) - 16 = 21 - 16 = 5$.

2. Given that $2x - 3 = 5 - x$, add x to both sides and then add 3 to both sides to get $3x = 8$. Now divide both sides by 3 to get $x = 8/3 = 2.\bar{6}$. Checking, we see that $2(8/3) - 3 = \frac{16}{3} - 3 = \frac{16}{3} - \frac{9}{3} = \frac{7}{3}$ and $5 - (8/3) = \frac{15}{3} - \frac{8}{3} = \frac{7}{3}$.

3. Given that $\frac{1}{2}(x - 3) + x = 17 + 3(4 - x)$, we first simplify the left and right hand sides using the distributive property to get $\frac{1}{2}x - \frac{3}{2} + x = 17 + 12 - 3x$. Combining like terms on both sides gives $\frac{3}{2}x - \frac{3}{2} = 29 - 3x$. Now we add $3x$ and $\frac{3}{2}$ to both sides, obtaining $\frac{9}{2}x = \frac{61}{2}$. Dividing both sides by $\frac{9}{2}$ (or multiplying both sides by $\frac{2}{9}$) gives $x = \frac{61}{2} \cdot \frac{2}{9} = \frac{61}{9}$. Checking we see that $LHS = \frac{1}{2}(\frac{61}{9} - \frac{27}{9}) + \frac{61}{9} = \frac{1}{2}\frac{34}{9} + \frac{61}{9} = \frac{17}{9} + \frac{61}{9} = \frac{78}{9}$ and $RHS = 17 + 3(\frac{36}{9} - \frac{61}{9}) = 17 + 3 \cdot \left(\frac{-25}{9}\right) = 17 - \frac{75}{9} = \frac{153}{9} - \frac{75}{9} = \frac{78}{9}$.

4. Given that $\frac{5}{x} = \frac{2}{x-3}$, we "cross-multiply" to obtain $5(x-3) = 2x$. Distributing the 5 gives $5x - 15 = 2x$. Subtracting $2x$ and adding 15 to both sides gives $3x = 15$. Dividing both sides by 3 gives $x = 5$. Checking, we see that $\frac{5}{5} = 1$ and $\frac{2}{5-3} = \frac{2}{2} = 1$. NOTE: Checking is *very* important in this kind of problem. When there are x's in the denominator of fractions in equations, it is possible that your final "solution" doesn't satisfy the original equation (because you would divide by zero)–so it is really *not* a solution to the original problem.

In the "multiply and simplify" problems, we must multiply each term in the left-hand factor with each term in the right-hand factor, and then simplify by combining like terms. In the case of $binomial \times binomial$, we can use the so-called FOIL method.

5. $(4x-1)(-3x+2) = -12x^2 + 8x + 3x - 2 = -12x^2 + 11x - 2$.

6. $(x-1)(x^2+x+1) = x^3 + x^2 + x - x^2 - x - 1 = x^3 - 1$.

7. $(x+1)(x^2-x+1) = x^3 - x^2 + x + x^2 - x + 1 = x^3 + 1$.

8. $(x-2)(x+2) = x^2 + 2x - 2x - 4 = x^2 - 4$.

9. $(x-2)(x-2) = x^2 - 2x - 2x + 4 = x^2 - 4x + 4$.

10. $(x^3+2x-1)(x^3-5x^2+4) = x^6 - 5x^5 + 4x^3 + 2x^4 - 10x^3 + 8x - x^3 + 5x^2 - 4 = x^6 - 5x^5 + 2x^4 - 7x^3 + 5x^2 + 8x - 4$

11. For a number x to be in the domain of the function $f(x) = \sqrt{1+x}$, we require $1 + x \geq 0$ (so we don't take the square root of a negative number). Subtracting 1 from both sides of this inequality yields $x \geq -1$. Thus, in interval notation, the domain is the set $[-1, \infty)$.

12. For a number x to be in the domain of the function $f(x) = \frac{1}{1+x}$, we require $1 + x \neq 0$ (so we don't divide by zero). Thus, we require $x \neq -1$. In interval notation, the domain is the set $(-\infty, -1) \cup (-1, \infty)$.

13. For a number x to be in the domain of the function $f(x) = \frac{1}{\sqrt{x}}$, we require $x > 0$ (so we don't divide by zero or take the square root of a negative number). In interval notation, the domain is the set $(0, \infty)$.

14. For a number x to be in the domain of the function $f(x) = \frac{1}{\sqrt{1+x}}$, we require $1 + x > 0$ (so we don't divide by zero or take the square root of a negative number). Subtracting 1 from both sides of this inequality yields $x > -1$. Thus, in interval notation, the domain is the set $(-1, \infty)$.

15. For a number x to be in the domain of the function $f(x) = \frac{1}{1+x^2}$, we require that $1 + x^2 \neq 0$ (so we don't divide by zero). But no matter what x is, $1 + x^2 > 0$. Therefore, the domain is $\mathbb{R}$, the set of all real numbers.

16. $f(3) = (3)^2 - 3(3) + 4 = 9 - 9 + 4 = 4$, $f(a) = a^2 - 3a + 4$, $f(-t) = (-t)^2 - 3(-t) + 4 = t^2 + 3t + 4$, and $f(x^2+1) = (x^2+1)^2 - 3(x^2+1) + 4 = x^4 + 2x^2 + 1 - 3x^2 - 3 + 4 = x^4 - x^2 + 2$.

By trial & error, we see that:

17. $x^2 - x - 20 = (x-5)(x+4)$

18. $x^2 - 10x + 21 = (x-3)(x-7)$

19. $x^2 + 10x + 16 = (x+8)(x+2)$

20. $x^2 + 8x - 105 = (x+15)(x-7)$

21. $4x^2 + 11x - 3 = (4x-1)(x+3)$

22. $-2x^2 + 7x + 15 = -(2x^2 - 7x - 15) = -(2x+3)(x-5)$

23. $x^2 - 2 = (x - \sqrt{2})(x + \sqrt{2})$ tricky, tricky, tricky!!! :)

24. $x^2 + 6x - 16 = 0$

(a) By factoring: $x^2 + 6x - 16 = 0$ implies that $(x + 8)(x - 2) = 0$. Thus, $x = -8$ or $x = 2$.

(b) By the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{36 - 4(1)(-16)}}{2(1)} = \frac{-6 \pm \sqrt{100}}{2} = \frac{-6 \pm 10}{2} = 2 \text{ or } -8$$

(c) By completing the square: by adding 16 to both sides of $x^2 + 6x - 16 = 0$, we get $x^2 + 6x = 16$. Now, adding $9 = 3^2 = (6/2)^2$ to both sides makes the left-hand side a perfect square: $x^2 + 6x + 9 = 25$. We can factor the left hand side to get $(x + 3)^2 = 25$. Now take the square root of both sides, allowing for the two square roots of 25 on the right hand side to obtain $x + 3 = \pm 5$. Subtracting 3 from both sides gives $x = -3 \pm 5$. In other words, $x = -8$ or $x = 2$.

Checking: $(-8)^2 + 6(-8) - 16 = 64 - 48 - 16 = 0$ and $(2)^2 + 6(2) - 16 = 4 + 12 - 16 = 0$.

25. $-x^2 - 3x - 2 = 0$

(a) By factoring: $-x^2 - 3x - 2 = 0$ implies that $x^2 + 3x + 2 = 0$ (multiply both sides by -1). Factoring gives $(x + 1)(x + 2) = 0$. Thus, $x = -1$ or $x = -2$.

(b) By the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3 \pm \sqrt{9 - 4(-1)(-2)}}{2(-1)} = \frac{3 \pm \sqrt{1}}{-2} = \frac{3 \pm 1}{-2} = -2 \text{ or } -1$$

(c) By completing the square: by adding 2 to both sides of $-x^2 - 3x - 2 = 0$, we get $-x^2 - 3x = 2$. Now, multiply both sides by -1 to get $x^2 + 3x = -2$. Now, adding $9/4 = (3/2)^2$ to both sides makes the left-hand side a perfect square: $x^2 + 3x + \frac{9}{4} = -2 + \frac{9}{4} = \frac{1}{4}$. We can factor the left hand side to get $(x + \frac{3}{2})^2 = \frac{1}{4}$. Now take the square root of both sides, allowing for the two square roots of 1/4 on the right hand side to obtain $x + \frac{3}{2} = \pm\frac{1}{2}$. Subtracting 3/2 from both sides gives $x = -\frac{3}{2} \pm \frac{1}{2}$. In other words, $x = -1$ or $x = -2$.

Checking: $-(-1)^2 - 3(-1) - 2 = -1 + 3 - 2 = 0$ and $-(-2)^2 - 3(-2) - 2 = -4 + 6 - 2 = 0$.

26. $2x^2 + 2x - 4 = 0$

(a) By factoring: $2x^2 + 2x - 4 = 0$ implies that $x^2 + x - 2 = 0$ (divide both sides by 2). Factoring gives $(x + 2)(x - 1) = 0$. Thus, $x = -2$ or $x = 1$.

(b) By the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{4 - 4(2)(-4)}}{2(2)} = \frac{-2 \pm \sqrt{36}}{4} = \frac{-2 \pm 6}{4} = 1 \text{ or } -2$$

(c) By completing the square: by adding 4 to both sides of $2x^2 + 2x - 4 = 0$, we get $2x^2 + 2x = 4$. Now divide both sides by 2 to give us $x^2 + x = 2$. Now, adding $\frac{1}{4} = (1/2)^2$ to both sides makes the left-hand side a perfect square: $x^2 + x + \frac{1}{4} = 2 + \frac{1}{4} = \frac{9}{4}$. We can factor the left hand side to get $(x + \frac{1}{2})^2 = \frac{9}{4}$. Now take the square root of both sides, allowing for the two square roots of $\frac{9}{4}$ on the right hand side to obtain $x + \frac{1}{2} = \pm\frac{3}{2}$. Subtracting 1/2 from both sides gives $x = -\frac{1}{2} \pm \frac{3}{2}$. In other words, $x = 1$ or $x = -2$.

Checking: $2(1)^2 + 2(1) - 4 = 2 + 2 - 4 = 0$ and $2(-2)^2 + 2(-2) - 4 = 8 - 4 - 40 = 0$.

27. Squaring both sides of $\sqrt{x} = \sqrt{2x - 1}$ gives $x = 2x - 1$. Solving this equation for x gives us $x = 1$. Checking in the *original* equation: $LHS = \sqrt{1} = 1$, $RHS = \sqrt{2(1) - 1} = \sqrt{1} = 1$.

28. Squaring both sides of $\sqrt{x^2 - 3} = \sqrt{2x}$ gives $x^2 - 3 = 2x$. Subtract $2x$ from both sides to get $x^2 - 2x - 3 = 0$. The left-hand side can now be factored to give $(x - 3)(x + 1) = 0$, so $x = 3$ or $x = -1$. **CHECKING in the ORIGINAL equation:** $LHS = \sqrt{(3)^2 - 3} = \sqrt{9 - 3} = \sqrt{6} = \sqrt{2(3)} = RHS$, however, if you plug $x = -1$ into either side of this equation, you get the square root of a negative number. Therefore, for us, $x = -1$ is not a solution (even though the LHS and RHS are equal the *imaginary* number $\sqrt{-2} = i\sqrt{2}$).

29. $|x-5| = 4$ implies that either $x-5=4$ or $x-5=-4$. Thus, either $x=9$ or $x=1$. Checking shows that both of these numbers are solutions: $|9-4| = |4| = 4$ and $|1-5| = |-4| = 4$.

30. Subtracting 4 from both sides of $2x+4 \geq 3$ gives $2x \geq -1$. Now divide both sides by 2 to get $x \geq -\frac{1}{2}$. The logic also works in the other direction: if $x \geq -\frac{1}{2}$, this will imply that $2x+4 \geq 3$. Thus, the solution set is the interval $[-\frac{1}{2}, \infty)$.

31. Subtracting 4 from both sides of $-2x+4 \geq 3$ gives $-2x \geq -1$. Now divide both sides by -2 and switch the direction of the inequality to get $x \leq \frac{1}{2}$. The logic also works in the other direction: if $x \leq \frac{1}{2}$, this will imply that $-2x+4 \geq 3$. Thus, the solution set is the interval $(-\infty, \frac{1}{2}]$.

32. Take the equation $\frac{x+4}{x-3} = 2$ and multiply both sides by $x-3$ to get $x+4 = 2(x-3)$. Now solve this equation: $x+4 = 2x-6 \Longrightarrow x = 10$. Checking: $\frac{10+4}{10-3} = 14/7 = 2$.

33. $x^2 - x - 2 > 0$ implies that $(x-2)(x+1) > 0$. Thus, either $x-2 > 0$ and $x+1 > 0$ OR $x-2 < 0$ and $x+1 < 0$. Thus, either $x > 2$ and $x > -1$ OR $x < 2$ and $x < -1$. Thus, either $x > 2$ OR $x < -1$. The logic also works in the other direction: if $x > 2$ OR $x < -1$, then $(x-2)(x+1) > 0$ so $x^2 - x - 2 > 0$. Thus, the solution set is $(-\infty, -1) \cup (2, \infty)$.

34. We need to get a common denominator. The simplest one to choose is $(x+2)(x-4)$. Multiplying the top and bottom of the first fraction by $x-4$ and multiplying the top and bottom of the second fraction by $x+2$ and combining the fractions produces:

$$\frac{x}{x+2} + \frac{3}{x-4} = \frac{x(x-4)}{(x+2)(x-4)} + \frac{3(x+2)}{(x+2)(x-4)} = \frac{(x^2-4x)+(3x+6)}{(x+2)(x-4)}$$

Simplifying the top and bottom gives us:

$$\frac{x^2 - x + 6}{x^2 - 2x - 8}$$

If there were a common factor on the top and bottom, we would cancel it out. However, there are no common factors. Therefore, this is our final answer.

35. We need to get a common denominator. The simplest one to choose is $(x-1)(x-2)(x-3)$. Multiplying the top and bottom of the first fraction by $x-3$ and multiplying the top and bottom of the second fraction by $(x-1)(x-2)$ and combining the fractions produces the following expressions (which are equal to the original):

$$\frac{(x^2+1)(x-3)}{(x-1)(x-2)(x-3)} - \frac{x^3(x-1)(x-2)}{(x-1)(x-2)(x-3)} = \frac{(x^3-3x^2+x-3)-(x^5-3x^4+2x^3)}{(x-1)(x-2)(x-3)}$$

Simplifying the top and bottom gives us:

$$\frac{-x^5+3x^4-x^3-3x^2+x-3}{(x-1)(x^2-5x+6)} = \frac{-x^5+3x^4-x^3-3x^2+x-3}{x^3-6x^2+11x-6}$$

Again, we would technically need to check for common factors to simplify this "completely". To do this, it is enough to determine whether 1, 2, or 3 are zeros of the polynomial in the numerator (since they are the zeros of the polynomial in the denominator). Let's call the numerator P, so $P(x) = -x^5+3x^4-x^3-3x^2+x-3$. $P(1) = -1+3-1-3+1-3 = -4 \neq 0$, $P(2) = -32+48-8-12+2-3 = -5 \neq 0$, and $P(3) = -243+243-27-27+3-3 = -54 \neq 0$. Therefore, there a no common factors, so the answer above is as simple as possible.

36. We can factor the top and bottom to obtain

$$\frac{x^2+x-2}{x^2-1} = \frac{(x+2)(x-1)}{(x+1)(x-1)}$$

There is a common factor of $x-1$, so we can cancel this to obtain our final answer:

$$\frac{x+2}{x+1}$$

It should be pointed out that this expression is equal to the first *as long as* $x \neq 1$.

37. We factor the numerator and denominator to obtain:

$$\frac{x^2+5x+6}{x^2-3x+2} = \frac{(x+3)(x+2)}{(x-1)(x-2)}$$

The numerator and denominator here have *no* common factors. Thus, this expression cannot be simplified.

38. The quickest approach here is to immediately multiply the top and bottom of this "double-decker fraction" by $(x+2)(x-1)$. Doing this, and canceling things, we obtain:

$$\frac{\left(\frac{x}{x+2}+3\right)}{\frac{x+1}{x-1}} \frac{(x+2)(x-1)}{(x+2)(x-1)} = \frac{x(x-1)+3(x+2)(x-1)}{(x+1)(x+2)}$$

Simplifying:

$$\frac{x^2-x+3x^2+3x-6}{x^2+3x+2} = \frac{4x^2+2x-6}{x^2+3x+2}$$

The top of this can be factored as $2(2x+3)(x-1)$, and so has no common factors with the bottom. Therefore, we are done.